R & D NOTES

A Mathematical Definition of Choking Phenomenon and a Mathematical Model for Predicting Choking Velocity and Choking Voidage

WEN-CHING YANG

Westinghouse Research Laboratories
Pittsburgh, Pennsylvania 15235

This note is intended to define choking in a vertical pneumatic conveying line mathematically and to present an unique approach to calculating the choking velocity and choking voidage by using readily available properties of the transported materials and system characteristics. The phenomenon of choking has been described in Zenz and Othmer (1960) and other standard textbooks on fluidization engineering. Understanding the choking phenomenon is essential in arriving at an optimum design. Unfortunately, little information is available in the literature for predicting the choking velocity and choking voidage. The difficulty stemmed partly from the fact that the point of choking was usually determined by subjective observation. As Capes and Nakamura (1973) pointed out, choking was not a clear-cut phenomenon but involved a whole range of instabilities. They defined the choking as the point where internal solid circulation, with solids moving downward at the wall and moving upward in the core, began. However, with dense particles like steel shots, they observed that the slugging phenomenon took place at gas velocities considerably above those at the point of internal solid circulation. Apparently, an objective definition of choking based on theoretical considerations is needed if a general predicting model is to be developed.

REVIEW OF PREVIOUS WORKS

Recently Leung et al. (1971) reviewed the existing correlations and pointed out the inadequacy of the correlations by Zenz and Othmer (1960) and by Doig and Roper (1963). In turn, they proposed a method for calculating solid flow rates at the onset of choking based on two assumptions: the voidage at the onset of choking ϵ_c is equal to a constant value of 0.97, and the slip velocity $U_{sl} = (U_f)_c - (U_p)_c$ at the onset of choking is equal to U_t, the terminal velocity of a single particle. The error in assuming $U_{sl} = U_t$ was likely to be small; however, the assumption of $\epsilon_c = 0.97$ involved too large an error. The experimental choking voidages ranged from 0.87 to 0.99 for smaller pipe sizes (Lewis et al., 1949; Ormiston, 1966). Assuming a constant value of 0.97 could give an error in calculating choking flow rates of up to 400%. For a larger pipe size, Capes and Nakamura (1973) reported a much larger choking voidage (up to 0.9994). The assumption of constant

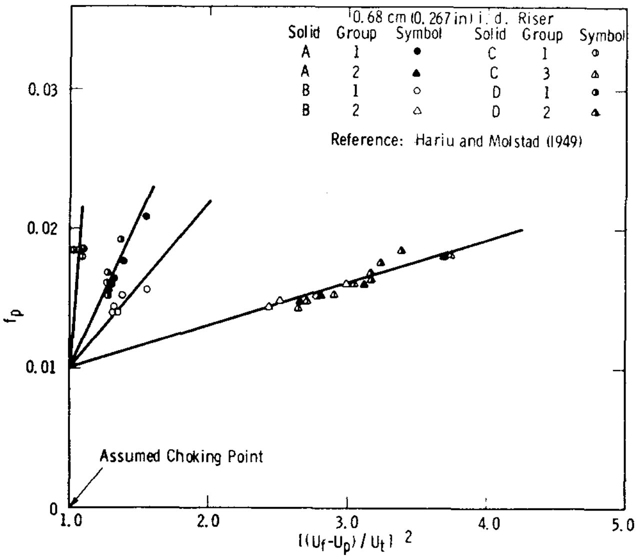

Fig. 1. Experimental evidence of constant solid friction factor at choking.

choking voidage of 0.97 can lead to an error of 5 000%! Despite this handicap, the approach was reasonable in light of the fact that all other methods in the literature for predicting choking flow rates were unsatisfactory.

MATHEMATICAL DEFINITION OF CHOKING

As mentioned previously, Leung et al. (1971) based their model on two assumptions; that is, $\epsilon_c = 0.97$ and $U_t = (U_f)_c - (U_p)_c$ at choking. It was known that experimental values of choking voidage ranged from 0.87 to 0.9994 and that choking voidage increased with increasing diameter and decreased with increasing choking velocity (Tables 1 to 3 in Supplement). An assumption of constant voidage at choking could not reconcile with the experimental findings. The present approach maintains the assumption that $U_t = (U_f)_c - (U_p)_c$, but, instead of assuming constant voidage, the solid friction factor f_p is assumed to be constant at choking. The experimental evidence which shows a constant f_p at choking is shown in Figure 1. Solid friction factors in a vertical pneumatic

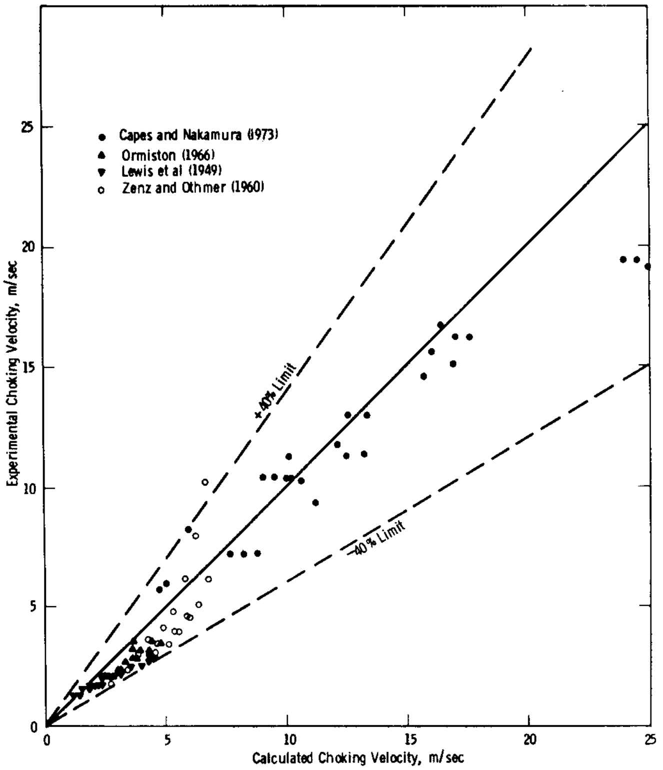

Fig. 2. Comparison of calculated and experimental choking velocities.

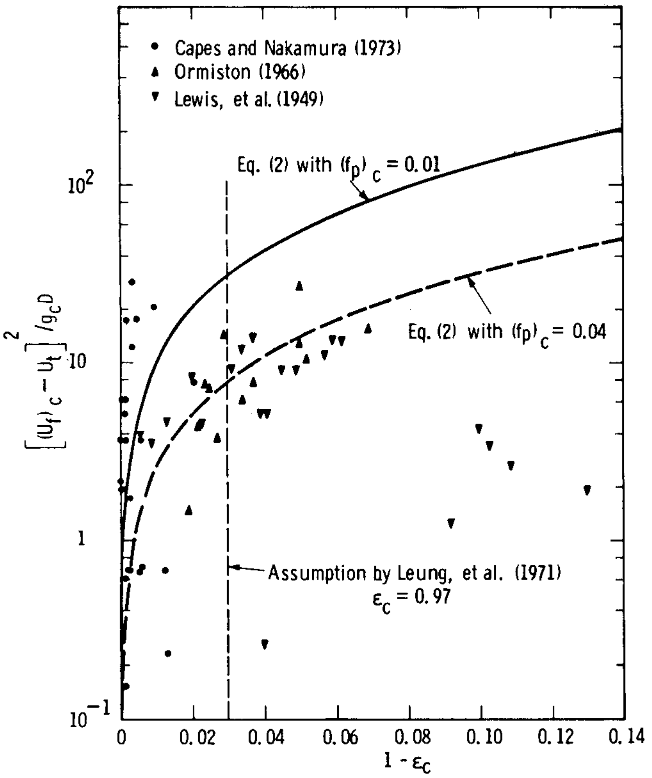

Fig. 3. Comparison of calculated and experimental choking voidages.

transport line of 0.68 cm I.D. (Hariu and Molstad, 1949) were plotted against the dimensionless group $[(U_f - U_p)/U_t]^2$. Extrapolation of the data to the choking point where $U_f - U_p = U_t$ converged to a common point of $f_p \cong 0.01$. Choking in vertical pneumatic conveying was thus defined mathematically as the point at which the solids friction factor is equal to 0.01.

Dynamic relationship between the solids and gases in a vertical pneumatic conveying line has been described earlier with the following modified terminal velocity equation by Yang (1973a):

$$U_p = U_f - U_t \cdot \sqrt{\left(1 + \frac{f_p U_p^2}{2g_c D}\right) \cdot \epsilon^{4.7}} \qquad (1)$$

At choking, Equation (1) can be simplified and rearranged to give

$$(f_p)_c = \frac{2g_c D\,(\epsilon^{-4.7} - 1)}{[(U_f)_c - U_t]^2} = 0.01 \qquad (2)$$

Equation (2) correctly predicts an increase of choking voidage with increasing diameter and a decrease of choking voidage with increasing choking velocity.

EVALUATION OF CHOKING VELOCITY AND CHOKING VOIDAGE

The carrying capacity of a vertical pneumatic transport tube at choking can be expressed as

$$G_c = (U_s)_c\, \rho_p = [(U_f)_c - U_t]\, \rho_p\, (1 - \epsilon_c) \qquad (3)$$

If the amount of solids to be transported is known, the choking velocity and choking voidage can be obtained by trial and error between Equations (2) and (3). The choking flow rate can then be calculated from Equation (3). If the operating gas velocity is known, the maximum carrying capacity of the transport line can be obtained by solving Equations (2) and (3) analytically.

COMPARISON WITH LITERATURE DATA

The proposed model was used to calculate the choking velocity and choking voidage from experimental data available in the literature. Comparison between the calculated and experimental choking velocities and voidages is summarized in Figures 2 and 3. The data included in this comparison have specific gravities ranging from 0.91 to 7.85, particle sizes from 40 to 3400μ, particle terminal velocities from 0.12 to 22.87 m/s and pipe sizes from 2.54 to 7.62 cm.

The voidage data by Zenz and Othmer (1960) were not included in this comparison because they obtained those data by calculation rather than through experimental measurement. Their data on choking velocity were included in Figure 2.

The calculated results were tabulated and compared with those obtained by Capes and Nakamura (1973), Ormiston (1966), and Lewis et al. (1949) in Tables 1 to 3, respectively, in the Supplement. The calculated values were shown for both $(f_p)_c = 0.01$ and $(f_p)_c = 0.02$ for comparison. Typical results are summarized in Table 1.

DISCUSSION

The model fitted the data by Capes and Nakamura (1973) surprisingly well. For choking velocity, it was good to ±20%, and for choking voidage, good to the third significant figure. Since the data by Capes and Nakamura (1973) were the most accurate (in author's opinion) and with widest material properties, the performance of the proposed model was gratifying. The model also fitted the data by Ormiston (1966) and Lewis et al. (1949) reasonably well. For $(f_p)_c = 0.01$, the model fitted the choking velocity data to ±30% for 90% of the data and to ±20% for 60% of the data, and for choking voidage, good to at least the second decimal point.

The assumption of $(f_p)_c = 0.04$ gave a much better correlation for Ormiston (1966) and Lewis et al. (1949)

TABLE 1. COMPARISON BETWEEN CALCULATED AND EXPERIMENTAL CHOKING VELOCITIES AND VOIDAGES

Material	Particle diameter, μ	Experimental Choking* velocity, m/s	Experimental Choking voidage	Calculated Choking velocity, m/s	Calculated Choking voidage	References
Steel ($\rho_p = 7.51$ g/cm^3,	260	8.26	0.9967	5.93	0.9946	Capes and Nakamura (1973)
$U_t = 3.99$ m/s)	260	5.64	0.9994	4.73	0.9992	
Steel ($\rho_p = 7.70$ g/cm^3,	2,340	19.02	0.9902	25.05	0.9924	Capes and Nakamura (1973)
$U_t = 22.93$ m/s)	2,340	19.36	0.9986	24.01	0.9982	
Polyethylene ($\rho_p = 0.91$ g/cm^3,	3,400	9.34	0.9923	11.22	0.9934	Capes and Nakamura (1973)
$U_t = 9.15$ m/s)	3,400	10.36	0.9991	10.05	0.9988	
Rape seed ($\rho_p = 1.09$ g/cm^3,	1,780	7.20	0.9876	8.80	0.9920	Capes and Nakamura (1973)
$U_t = 6.49$ m/s)	1,780	7.20	0.9975	7.66	0.9978	
Glass ($\rho_p = 2.86$ g/cm^3,	2,900	16.16	0.9938	17.63	0.9924	Capes and Nakamura (1973)
$U_t = 15.43$ m/s)	2,900	16.69	0.9994	16.42	0.9985	
Glass beads ($\rho_p = 2.48$ g/cm^3,	40	1.22	0.994	1.16	0.9962	Lewis et al. (1949)
$U_t = 0.12$ m/s)	40	2.01	0.966	2.33	0.9834	
Sand ($\rho_p = 2.66$ g/cm^3,	120	2.36	0.975	3.15	0.9818	Ormiston (1966)
$U_t = 1.10$ m/s)	120	3.51	0.950	3.69	0.9713	

* Choking velocity here refers to superficial velocity at choking.

voidage data than that at $(f_p)_c = 0.01$ (see Figure 3). The difficulty of determining solid friction factors in a pneumatic conveying line has been discussed previously (Yang, 1973*a*, 1973*b*, 1974*a*, 1974*b*). Extrapolation of experimental solid friction factors to the choking point will be somewhat different, using different sets of experimental data from different researchers; however, it seems clear that the assumption of constant solid friction factor at choking is reasonable. Moreover, the proposed model, which gives much better predictions for choking velocities than any other model currently available in the literature, is also the only model available for predicting the choking voidage from the properties of the transported materials and the system characteristics. The model should be valuable for design purpose.

ACKNOWLEDGMENT

Encouragement from Dr. D. L. Keairns is gratefully acknowledged. This work was performed as part of the Westinghouse Coal Gasification Program supported jointly by Energy Research and Development Administration and Westinghouse.

NOTATION

A = cross-sectional area of conveying line, cm^2
D = diameter of conveying line, cm
d_p = particle diameter, cm
f_p = solid friction factor
g = gravity acceleration, cm/s^2
G = solid flow rate per unit area, g/s-cm^2
R = loading ratio, W_s/W_f
U_0 = superficial gas velocity, cm/s
U_f = actual gas velocity; $U_f = U_0/\epsilon$, cm/s
U_p = actual solid particle velocity; $U_p = U_s/(1-\epsilon)$, cm/s
U_s = superficial solid velocity, cm/s
U_{sl} = slip velocity; $U_{sl} = U_f - U_p$, cm/s
U_t = terminal velocity of a single particle, cm/s
W_f = gas flow rate, g/s
W_s = solid flow rate, g/s
ρ_p = particle density, g/cm^3
ϵ = voidage

Subscripts

c = at choking

LITERATURE CITED

Capes, C. E., and K. Nakamura, "Vertical Pneumatic Conveying—An Experimental Study with Particles in the Intermediate and Turbulent Flow Regimes," *Can. J. Chem. Eng.*, **51**, No. 2, 31 (1973).

Doig, I. D., and G. H. Roper, "The Minimum Gas Rate for Dilute Phase Solids Transportation in a Gas Stream," *Aust. Chem. Eng.*, **4**, No. 1, 9 (1963).

Hariu, O. H., and M. C. Molstad, "Pressure Drop in Verticle Tubes in Transport of Solids by Gases," *Ind. Eng. Chem.*, **41**, No. 6, 1148 (1949).

Leung, L. S., R. J. Wiles, and D. J. Nicklin, "Correlation for Predicting Choking Flowrates in Vertical Pneumatic Conveying," *Ind. Eng. Chem. Process Design Develop.*, **10**, No. 2, 183 (1971).

Lewis, W. K., E. R. Gilliland, and W. C. Bauer, "Characteristics of Fluidized Particles," *Ind. Eng. Chem.*, **41**, 1104 (1949).

Ormiston, R. M., Ph.D. thesis, Cambridge University, England (1966).

Yang, W. C., "Estimating the Solid Particle Velocity in Vertical Pneumatic Conveying Lines," *Ind. Eng. Chem. Fundamentals*, **12**, No. 3, 349 (1973*a*).

———., D. L. Keairns, and D. H. Archer, "Estimating the Solid Particle Velocity in Horizontal Pneumatic Conveying Lines," *Can. J. Chem. Eng.*, **51**, 779 (1973*b*).

———, "Correlations for Solid Friction Factors in Vertical and Horizontal Pneumatic Conveyings," *AIChE J.*, **20**, No. 3, 605 (1974*a*).

———., "Additional Notes on Correlations for Solid Friction Factors in Pneumatic Conveyings," paper presented at the 77th AIChE Meeting, Pittsburgh, Pa. (June, 1974*b*).

Zenz, F. A., and D. F. Othmer, *Fluidization and Fluid-Particle Systems*, Reinhold, New York (1960).

Supplementary material has been deposited as Document No. 02645 with the National Auxiliary Publications Service (NAPS), c/o Microfiche Publications, 440 Park Ave. South, New York, NY 10016 and may be obtained for $1.50 for microfiche or $5.00 for Photocopies.

Manuscript received February 20, 1975; revision received June 3, and accepted June 6, 1975.